

OSCILLATION OF NONLINEAR DIFFERENCE EQUATIONS WITH DELAYED ARGUMENT

S. PINELAS¹ AND R. KOPLATADZE²

¹Departamento de Matematica, Universidade dos Acores
Portugal

E-mail: sandra.pinelas@clix.pt

²Department of Mathematics of Tbilisi State University
University st. 2, Tbilisi 0186, Georgia

E-mail: r.koplatadze@yahoo.com

ABSTRACT. The following difference equation with delayed argument

$$\Delta^2 u(k) + F(k, u(\tau(k))) = 0$$

is considered, where $F : N \times R \rightarrow R$, $\tau : N \rightarrow N$, $\tau(k) \leq k$ for $k \in N$, $\lim_{k \rightarrow +\infty} \tau(k) = +\infty$ and $\Delta u(k) = u(k+1) - u(k)$, $\Delta^2 = \Delta \circ \Delta$. In the paper sufficient (necessary and sufficient) conditions are established for all proper solutions of the above equation to be oscillatory.

AMS (MOS) Subject Classification. 34K06, 34K11.

1. INTRODUCTION

Consider the difference equation

$$\Delta^2 u(k) + F(k, u(\tau(k))) = 0, \quad (1.1)$$

where $F : N \times R \rightarrow R$, $\tau : N \rightarrow N$, $\Delta u(k) = u(k+1) - u(k)$ and $\Delta^2 = \Delta \circ \Delta$. Everywhere it will be assumed that

$$F(k, x) \operatorname{sign} x \geq 0 \quad \text{for } k \in N \quad \text{and } x \in R, \quad (1.2)$$

$$\tau(k) \leq k \quad \text{for } k \in N, \quad \lim_{k \rightarrow +\infty} \tau(k) = +\infty \quad (1.3)$$

and

$$\sup \{|F(i, x)| : i \geq k\} > 0 \quad \text{for } k \in N, \quad x \neq 0. \quad (1.4)$$

For any $n \in N$, denote $N_n = \{n, n+1, \dots\}$.

Definition 1.1. For $n \in N$ put $n_0 = \min\{\tau(k) : k \in N_n\}$. A function $u : N_{n_0} \rightarrow R$ is said to be a proper solution of (1.1), if it satisfies (1.1) on N_n and

$$\sup \{|u(i)| : i \geq k\} > 0 \quad \text{for } k \in N.$$